M.Sc. Program

Two years M.Sc. Mathematics program consists of two parts namely Part-I and Part II. The regulation, Syllabi and Courses of Reading for the M.Sc. (Mathematics) Part-I and Part-II Scheme are given below.

Regulations

The following regulations will be observed by M.Sc. (Mathematics) Private students

i. There are a total of 1200 marks for M.Sc. (Mathematics) for Private students as is the case with other M.Sc. subjects.
ii. There are five papers in Part-I and six papers in Part-II. Each paper carries 100 marks.
iii. There is a Viva Voce Examination at the end of M.Sc. Part II. The topics of Viva Voce Examination shall be from the following courses of M.Sc. Part-I (carrying 100 marks):
    a) Real Analysis
    b) Algebra
    c) Complex Analysis
    d) Differential Equation
    e) Topology and Functional Analysis

M.Sc. Part-I

The following five papers shall be studied in M.Sc. Part-I:

Paper I	Real Analysis
Paper II	Algebra
Paper III	Complex Analysis and Differential Geometry
Paper IV	Mechanics
Paper V	Topology and Functional Analysis

Note: All the papers of M.Sc. Part-I given above are compulsory.

M.Sc. Part-II

In M.Sc. Part-II examinations, there are six written papers. The following three papers are compulsory. Each paper carries 100 marks.

Paper I	Advanced Analysis
Paper II	Differential Equation
Paper III	Numerical Analysis

Optional Papers

A student may select any three of the following optional courses:

Paper IV-VI option (i)	Mathematical Statistics
Paper IV-VI option (ii)	Methods of Mathematical Physics
Paper IV-VI option (iii)	Group Theory
Paper IV-VI option (iv)	Rings and Modules
Paper IV-VI option (v)	Number Theory
Paper IV-VI option (vi)	Fluid Mechanics
Paper IV-VI option (vii)	Special Theory of Relativity and Analytical Mechanics
Paper IV-VI option (viii)	Theory of Approximation and Splines
Paper IV-VI option (ix)	Advanced Functional Analysis
Paper IV-VI option (x)	Theory of Optimization

Detailed Outline of Courses

M.Sc. Part I Papers

Paper I: Real Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Real Number System

Ordered sets, Fields, Completeness property of real numbers
The extended real number system, Euclidean spaces

Sequences and Series

Sequences, Subsequences, Convergent sequences, Cauchy sequences
Monotone and bounded sequences, Bolzano Weierstrass theorem
Series, Convergence of series, Series of non-negative terms, Cauchy condensation test
Partial sums, The root and ratio tests, Integral test, Comparison test
Absolute and conditional convergence

Limit and Continuity

The limit of a function, Continuous functions, Types of discontinuity
Uniform continuity, Monotone functions

Differentiation

The derivative of a function
Mean value theorem, Continuity of derivatives
Properties of differentiable functions.

Functions of Several Variables

Partial derivatives and differentiability, Derivatives and differentials of composite functions
Change in the order of partial derivative, Implicit functions, Inverse functions, Jacobians
Maxima and minima, Lagrange multipliers

Section-II (4/9)

The Riemann-Stieltjes Integrals

Definition and existence of integrals, Properties of integrals
Fundamental theorem of calculus and its applications
Change of variable theorem
Integration by parts

Functions of Bounded Variation

Definition and examples

Properties of functions of bounded variation

Improper Integrals

Types of improper integrals

Tests for convergence of improper integrals Beta and gamma functions

Absolute and conditional convergence of improper integrals

Sequences and Series of Functions

Definition of point-wise and uniform convergence

Uniform convergence and continuity

Uniform convergence and integration

Uniform convergence and differentiation

Recommended Books

1. W. Rudin, *Principles of Mathematical Analysis*, (McGraw Hill, 1976)
2. R. G. Bartle, *Introduction to Real Analysis*, (John Wiley and Sons, 2000)
3. T. M. Apostol, *Mathematical Analysis*, (Addison-Wesley Publishing Company, 1974)
4. A. J. Kosmala, *Introductory Mathematical Analysis*, (WCB Company , 1995)
5. W. R. Parzynski and P. W. Zipse, *Introduction to Mathematical Analysis*, (McGraw Hill Company, 1982)
6. H. S. Gaskill and P. P. Narayanaswami, *Elements of Real Analysis*, (Printice Hall, 1988)

Paper II: Algebra (Group Theory and Linear Algebra)

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Groups

Definition and examples of groups

Subgroups lattice, Lagrange’s theorem

Cyclic groups

Groups and symmetries, Cayley’s theorem

Complexes in Groups

Complexes and coset decomposition of groups

Centre of a group

Normalizer in a group

Centralizer in a group

Conjugacy classes and congruence relation in a group

Normal Subgroups

Normal subgroups

Proper and improper normal subgroups
Factor groups
Isomorphism theorems
Automorphism group of a group
Commutator subgroups of a group

Permutation Groups

Symmetric or permutation group
Transpositions
Generators of the symmetric and alternating group
Cyclic permutations and orbits, The alternating group
Generators of the symmetric and alternating groups

Sylow Theorems

Double cosets
Cauchy's theorem for Abelian and non-Abelian group
Sylow theorems (with proofs)
Applications of Sylow theory
Classification of groups with at most 7 elements

Section-II (4/9)

Ring Theory

Definition and examples of rings
Special classes of rings
Fields
Ideals and quotient rings
Ring Homomorphisms
Prime and maximal ideals
Field of quotients

Linear Algebra

Vector spaces, Subspaces
Linear combinations, Linearly independent vectors
Spanning set
Bases and dimension of a vector space
Homomorphism of vector spaces
Quotient spaces

Linear Mappings

Mappings, Linear mappings
Rank and nullity
Linear mappings and system of linear equations
Algebra of linear operators
Space L(X, Y) of all linear transformations

Matrices and Linear Operators

Matrix representation of a linear operator

Change of basis
Similar matrices
Matrix and linear transformations
Orthogonal matrices and orthogonal transformations
Orthonormal basis and Gram Schmidt process

Eigen Values and Eigen Vectors

Polynomials of matrices and linear operators
Characteristic polynomial
Diagonalization of matrices

Recommended Books

1. J. Rose, *A Course on Group Theory*, (Cambridge University Press, 1978)
2. I. N. Herstein, *Topics in Algebra*, (Xerox Publishing Company, 1964)
3. G. Birkhoff and S. Maclane, *A Survey of Modern Algebra*, (Macmillan, 1964)
4. Seymour Lipschutz, *Linear Algebra*, (McGraw Hill Book Company, 2001)
5. Humphreys, John F. *A Course on Group Theory*, (Oxford University Press, 2004)
6. P. M. Cohn, *Algebra*, (John Wiley and Sons, 1974)
7. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Pearson Education, 2002)

Paper III: Complex Analysis and Differential Geometry

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

The Concept of Analytic Functions

Complex numbers, Complex planes, Complex functions
Analytic functions
Entire functions
Harmonic functions
Elementary functions: Trigonometric, Complex exponential, Logarithmic and hyperbolic functions

Infinite Series

Power series, Derived series, Radius of convergence
Taylor series and Laurent series

Conformal Representation

Transformation, conformal transformation Linear transformation
Möbius transformations

Complex Integration

Complex integrals
Cauchy-Goursat theorem

Cauchy's integral formula and their
consequences Liouville's theorem
Morera's theorem
Derivative of an analytic function

Singularity and Poles

Review of Laurent series
Zeros, Singularities
Poles and residues
Cauchy's residue theorem
Contour Integration

Expansion of Functions and Analytic Continuation

Mittag-Leffler theorem
Weierstrass's factorization theorem
Analytic continuation

Section-II (4/9)

Theory of Space Curves

Introduction, Index notation and summation convention
Space curves, Arc length, Tangent, Normal and binormal
Osculating, Normal and rectifying planes
Curvature and torsion
The Frenet-Serret theorem
Natural equation of a curve
Involutes and evolutes, Helices
Fundamental existence theorem of space curves

Theory of Surfaces

Coordinate transformation
Tangent plane and surface normal
The first fundamental form and the metric
tensor The second fundamental form
Principal, Gaussian, Mean, Geodesic and normal curvatures
Gauss and Weingarten equations
Gauss and Codazzi equations

Recommended Books

1. H. S. Kasana, *Complex Variables*: *Theory and Applications*, (Prentice Hall, 2005)
2. M. R. Spiegel, *Complex Variables*, (McGraw Hill Book Company, 1974)
3. J. W. Brown, R. V. Churchill, *Complex Variables and Applications*, (McGraw Hill, 2009)
4. Louis L. Pennisi, *Elements of Complex Variables*, (Holt, Linehart and Winston, 1976)
5. W. Kaplan, *Introduction to Analytic Functions*, (Addison-Wesley, 1966)

6. R. S. Millman and G.D. Parker, *Elements of Differential Geometry,* (Prentice-Hall, 1977)
7. E. Kreyzig, *Differential Geometry,* (Dover Publications, 1991)
8. M. M. Lipschutz, *Schaum's Outline of Differential Geometry,* (McGraw Hill, 1969)
9. D. Somasundaram, *Differential Geometry,* (Narosa Publishing House, 2005)

Paper IV: Mechanics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Vector Integration

Line integrals

Surface area and surface integrals

Volume integrals

Integral Theorems

Green's theorem

Gauss divergence

theorem Stoke's theorem

Curvilinear Coordinates

Orthogonal coordinates

Unit vectors in curvilinear systems

Arc length and volume elements

The gradient, Divergence and curl

Special orthogonal coordinate systems

Tensor Analysis

Coordinate transformations

Einstein summation convention

Tensors of different ranks

Contravariant, Covariant and mixed tensors

Symmetric and skew symmetric tensors

Addition, Subtraction, Inner and outer products of tensors

Contraction theorem, Quotient law

The line element and metric tensor

Christoffel symbols

Section-II (4/9)

Non Inertial Reference Systems

Accelerated coordinate systems and inertial forces

Rotating coordinate systems

Velocity and acceleration in moving system: Coriolis, Centripetal and transverse acceleration

Dynamics of a particle in a rotating coordinate system

Planar Motion of Rigid Bodies

Introduction to rigid and elastic bodies, Degrees of freedom, Translations, Rotations, instantaneous axis and center of rotation, Motion of the center of mass

Euler's theorem and Chasle's theorem

Rotation of a rigid body about a fixed axis: Moments and products of inertia of various bodies including hoop or cylindrical shell, circular cylinder, spherical shell

Parallel and perpendicular axis theorem

Radius of gyration of various bodies

Motion of Rigid Bodies in Three Dimensions

General motion of rigid bodies in space: Moments and products of inertia, Inertia matrix

The momental ellipsoid and equimomental systems

Angular momentum vector and rotational kinetic energy

Principal axes and principal moments of inertia

Determination of principal axes by diagonalizing the inertia matrix

Euler Equations of Motion of a Rigid Body

Force free motion

Free rotation of a rigid body with an axis of symmetry

Free rotation of a rigid body with three different principal moments

Euler's Equations

The Eulerian angles, Angular velocity and kinetic energy in terms of Euler angles, Space cone

Motion of a spinning top and gyroscopes- steady precession, Sleeping top

Recommended Books

1. G. E. Hay, *Vector and Tensor Analysis*, (Dover Publications, Inc., 1979)
2. G. R. Fowles and G. L. Cassiday, *Analytical Mechanics*, (Thomson Brooks/Cole, 2005)
3. H. Goldstein, C. P. Poole and J. L. Safko, *Classical Mechanics*, (Addison-Wesley Publisihng Co., 2001)
4. M. R. Spiegel, *Theoretical Mechanics*, (McGraw Hill Book Company, 1980)
5. M. R. Spiegel, *Vector Analysis*, (McGraw Hill Book Company, 1981)
6. D. C. Kay, *Tensor Calculus*, (McGraw Hill Book Company, 1988)
7. E. C. Young, *Vector and Tensor Analysis*, (Marcel Dekker, Inc., 1993)
8. L. N. Hand and J. D. Finch, *Analytical Mechanics*, (Cambridge University Press, 1998)

Paper V: Topology & Functional Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Topology

Definition and examples

Open and closed sets

Subspaces
Neighborhoods
Limit points, Closure of a set
Interior, Exterior and boundary of a set

Bases and Sub-bases

Base and sub bases
Neighborhood bases
First and second axioms of countablility
Separable spaces, Lindelöf spaces
Continuous functions and homeomorphism
Weak topologies, Finite product spaces

Separation Axioms

Separation axioms
Regular spaces
Completely regular
spaces Normal spaces

Compact Spaces

Compact topological spaces
Countably compact spaces
Sequentially compact spaces

Connectedness

Connected spaces, Disconnected
spaces Totally disconnected spaces
Components of topological spaces

Section-II (5/9)

Metric Space

Review of metric spaces
Convergence in metric spaces
Complete metric spaces
Completeness proofs
Dense sets and separable spaces
No-where dense sets
Baire category theorem

Normed Spaces

Normed linear spaces
Banach spaces
Convex sets
Quotient spaces
Equivalent norms
Linear operators
Linear functionals
Finite dimensional normed spaces

Continuous or bounded linear operators
Dual spaces

Inner Product Spaces

Definition and examples
Orthonormal sets and
bases Annihilators,
Projections Hilbert space
Linear functionals on Hilbert spaces
Reflexivity of Hilbert spaces

Recommended Books

1. J. Dugundji, *Topology*, (Allyn and Bacon Inc., 1966)
2. G. F. Simmon, *Introduction to Topology and Modern Analysis*, (McGraw Hill Book Company, 1963)
3. Stephen Willard, *General Topology*, (Addison-Wesley Publishing Co., 1970)
4. Seymour Lipschutz, *General Topology*, (Schaum's Outline Series, McGraw Hill Book Company, 2004)
5. E. Kreyszig, *Introduction to Functional Analysis with Applications*, (John Wiley and Sons, 2006)
6. A. L. Brown and A. Page, *Elements of Functional Analysis*, (Van Nostrand Reinhold, 1970)
7. G. Bachman and L. Narici, *Functional Analysis*, (Academic Press, 1966)
8. F. Riesz and B. Sz. Nagay, *Functional Analysis*, (Dover Publications, Inc., 1965)

M.Sc. Part II Papers

Paper I: Advanced Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Advanced Set Theory

- Equivalent Sets
  - Countable and Uncountable Sets
  - The concept of a cardinal number
  - The cardinals $_0$ and c
- Addition and multiplication of cardinals
  - Cartesian product, Axiom of Choice, Multiplication of cardinal numbers
  - Order relation and order types, Well ordered sets, Transfinite induction
  - Addition and multiplication of ordinals
- Statements of Zorn's lemma, Maximality principle and their simple implications

Section-II (5/9)

Measure Theory

Outer measure, Lebesgue Measure, Measureable Sets and Lebesgue measure, Non measurable sets, Measureable functions

The Lebesgue Integral

The Rieman Integral, The Lebesgue integral of a bounded function The general Lebesgue integral

General Measure and Integration

Measure spaces, Measureable functions, Integration, General convergence theorems

Signed measures, The Lp-spaces, Outer measure and measurability The extension theorem

The Lebesgue Stieltjes integral, Product measures

Recommended Books

1. D. Smith, M. Eggen and R. ST. Andre, *A transition to Advanced Mathematics*, (Brooks Cole, 2004)
2. Seymour Lipschutz, *Set Theory and Related Topics*, (McGraw Hill, 1964)
3. Frankel, A. *Abstract Set theory*, (North Holland Publishing Co., 1961)
4. Royden, H. L. *Real Analysis*, (Prentice Hall, 1988)
5. Suppes, P. *Axiomatic Set Theory*, (Dover Publications Inc.,May 1973)
6. Halmos, P. R. *Naive Set Theory*, (Springer, 1974)
7. Halmos, P. R. *Measure Theory*, (Springer, 1974)
8. Rudin, W. *Real and Complex Analysis*, (McGraw-Hill Higher Education, 1987)

Paper II: Differential Equation (Ordinary and Partial Differential Equation)

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

First Order Ordinary Differential Equation

Basic concepts, Formation and solution of differential equations,
Separation of variables,
Homogeneous equations,
Exact equations,
Solution of linear equations by integrating factor,
Some special non-linear first order differential equations like Bernoulli's equations Ricatti equations and Clairaut equations

System of Ordinary Differential Equation

Basic theory of system of first order linear differential equations,
Homogeneous linear system with constant coefficients

Second and Higher Order Differential Equation

Initial value and boundary value problems
Linearly independence and Wronskian
Superposition principle
Homogeneous and non-homogeneous equations
Reduction of order
Solution of homogeneous equations with constant coefficients
Particular solution of non-homogeneous equations
Method of Undetermined coefficients
Variation of Parameters and Cauchy-Euler equations

Section-II (4/9)

First Order Partial Differential Equation

Formation of PDEs
Solutions of First Order PDEs
The Cauchy's problem for Quasi linear first order PDEs
First order nonlinear equations
Special types of first order equations

Second Order Partial Differential Equation

Basic concepts and definitions
Mathematical problems
Linear operators
Superposition
Canonical form: Hyperbolic, Parabolic and Elliptic equations,
PDEs of second order in two independent variables with constant and variable coefficients
Cauchy's problem for second order PDEs in two independent variables
Laplace equation, Wave equation, Heat equation

Methods of separation of variables

Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates

Recommended Books

1 William E. Boyce and Richard C. Diprima, *Elementary differential equations and boundary value problems*, (Seventh Edition John Wiley & Sons, Inc)
2 V. I. Arnold, *Ordinary Differential Equations*, (Springer, 1991)

3 Dennis G. Zill, Michael R. Cullen, *Differential equations with boundary value problems*, (Brooks Cole, 2008)
4 J. Wloka, *Partial Differential Equations*, (Cambridge University press, 1987)

Paper III: Numerical Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Error Analysis

Errors, Absolute errors, Rounding errors, Truncation errors
Inherent Errors, Major and Minor approximations in numbers

The Solution of Linear Systems

Gaussian elimination method with pivoting, LU Decomposition methods, Algorithm and convergence of Jacobi iterative Method, Algorithm and convergence of Gauss Seidel Method

Eigenvalue and eigenvector, Power method

The Solution of Non-Linear Equation

Bisection Method, Fixed point iterative method, Newton Raphson method, Secant method, Method of false position, Algorithms and convergence of these methods

Difference Operators

Shift operators
Forward difference operators
Backward difference operators
Average and central difference operators

Ordinary Differential Equations

Euler's, Improved Euler's, Modified Euler's methods with error analysis
Runge-Kutta methods with error analysis
Predictor-corrector methods for solving initial value problems
Finite Difference, Collocation and variational methods for boundary value problems

Section-II (4/9)

Interpolation

Lagrange's interpolation
Newton's divided difference interpolation
Newton's forward and backward difference interpolation, Central difference interpolation
Hermit interpolation
Spline interpolation
Errors and algorithms of these interpolations

Numerical Differentiation

Newton's Forward, Backward and central formulae for numerical differentiation

Numerical Integration

Rectangular rule
Trapezoidal rule
Simpson rule
Boole's rule
Weddle's rule
Gaussian quadrature formulae
Errors in quadrature formulae
Newton-Cotes formulae

Difference Equations

Linear homogeneous and non-homogeneous difference equations with constant coefficients

Recommended Books

1. Curtis F. Gerald and Patrick O. Wheatley, *Applied Numerical Analysis*, (Addison-Wesley Publishing Co. Pearson Education, 2003)
2. Richard L. Burden and J. Douglas Faires, *Numerical Analysis*, (Brooks/Cole Publishing Company,1997)
3. John H. Mathews, *Numerical Methods for Mathematics, Science and Engineering*, (Prentice Hall International, 2003)
4. Steven C. Chapra and Raymond P. Canale, *Numerical Methods for Engineers*, (McGraw Hill International Edition, 1998)

Paper (IV-VI) option (i): Mathematical Statistics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Probability Distributions

The postulates of probability
Some elementary theorems
Addition and multiplication rules
Baye's rule and future Baye's theorem
Random variables and probability functions

Discrete Probability Distributions

Uniform, Bernoulli and binomial distribution
Hypergeometric and geometric distribution
Negative binomial and Poisson distribution

Continuous Probability Distributions

Uniform and exponential distribution
Gamma and beta distributions
Normal distribution

Mathematical Expectations

Moments and moment generating functions
Moments of binomial, Hypergeometric, Poisson, Gamma, Beta and normal distributions

Section-II (5/9)

Functions of Random Variables

Distribution function technique

Transformation technique: One variable, Several variables

Moment-generating function technique

Sampling Distributions

The distribution of mean and variance

The distribution of differences of means and variances

The Chi-Square distribution

The t distribution

The F distribution

Regression and Correlation

Linear regression

The methods of least squares

Normal regression analysis

Normal correlation analysis

Multiple linear regression (along with matrix notation)

Recommended Books

1. J. E. Freund, *Mathematical Statistics*, (Prentice Hall Inc., 1992)
2. Hogg and Craig, *Introduction to Mathematical Statistics*, (Collier Macmillan, 1958)
3. Mood, Greyill and Boes, *Introduction to the Theory of Statistics*, (McGraw Hill)
4. R. E. Walpole, *Introduction to Statistics*, (Macmillan Publishing Company, 1982)
5. M. R. Spiegel and L. J. Stephens, *Statistics*, (McGraw Hill Book Company, 1984)

Paper (IV-VI) option (ii): Methods of Mathematical Physics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Sturm Liouville Systems

Some properties of Sturm-Liouville equations

Regular, Periodic and singular Sturm-Liouville systems and its applications

Series Solutions of Second Order Linear Differential Equations

Series solution near an ordinary point

Series solution near regular singular points

Series Solution of Some Special Differential Equations

Hypergeometric function $F(a, b, c; x)$ and its evaluation

Series solution of Bessel equation

Expression for $J_n(X)$ when n is half odd integer, Recurrence formulas for $J_n(X)$

Orthogonality of Bessel functions

Series solution of Legendre equation

Introduction to PDEs

Review of ordinary differential equation in more than one variables

Linear partial differential equations (PDEs) of the first order

Cauchy's problem for quasi-linear first order PDEs

PDEs of Second Order

PDEs of second order in two independent variables with variable coefficients

Cauchy's problem for second order PDEs in two independent variables

Boundary Value Problems

Laplace equation and its solution in Cartesian, Cylindrical and spherical polar coordinates

Dirichlet problem for a circle

Poisson's integral for a circle

Wave equation

Heat equation

Section-II (4/9)

Fourier Methods

The Fourier transform

Fourier analysis of generalized functions

The Laplace transform

Green's Functions and Transform Methods

Expansion for Green's

functions Transform methods

Closed form of Green's functions

Variational Methods

Euler-Lagrange equations

Integrand involving one, two, three and n variables

Necessary conditions for existence of an extremum of a function

Constrained maxima and minima

Recommended Books

1. D.G. Zill and M.R. Cullen, *Advanced Engineering Mathematics,* (Jones and Bartlett Publishers, 2006)
2. W.E. Boyce and R. C. Diprima, *Elementary Differential Equations and Boundary Value Problems,* (John Wiley & Sons, 2005)
3. E.T. Whittaker, and G. N. Watson, *A Course of Modern Analysis*, (Cambridge University Press, 1962)
4. I.N. Sneddon, *Elements of Partial Differential Equations,* (Dover Publishing, Inc., 2006)
5. R. Dennemyer, *Introduction to Partial Differential Equations and Boundary Value Problems,* (McGraw Hill Book Company, 1968)
6. D.L. Powers, *Boundary Value Problems and Partial Differential Equations,* (Academic Press, 2005)
7. W.E. Boyce, *Elementary Differential Equations,* (John Wiley & Sons, 2008)
8. M.L. Krasnov, G.I. Makarenko and A.I. Kiselev, *Problems and Exercises in the Calculus of Variations*, (Imported Publications, Inc., 1985)
9. J. Brown and R. Churchill, *Fourier Series and Boundary Value Problems,* (McGraw Hill, 2006)

Paper (IV-VI) option (iii): Advanced Group Theory

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

The Orbit Stablizer Theorem

Stablizer, Orbit, A group with p^2 elements Simplicity of A_n, n 5

Classification of Groups with at most 8 elements

Sylow Theorems

Sylow theorems (with proofs)

Applications of Sylow Theory

Products in Groups

Direct Products

Classification of Finite Abelian Groups

Characteristic and fully invariant subgroups

Normal products of groups

Holomorph of a group

Section-II (5/9)

Series in Groups

Series in groups

Zassenhaus lemma

Normal series and their refinements

Composition series

The Jordan Holder Theorem

Solvable Groups

Solvable groups, Definition and examples

Theorems on solvable groups

Nilpotent Groups

Characterisation of finite nilpotent groups Frattini subgroups

Extensions

Central extensions

Cyclic extensions

Groups with at most 31 elements

Linear Groups

Linear groups, types of linear groups

Representation of linear groups

The projective special linear groups

Recommended Books

1. J. Rotman, *The Theory of Groups*, (Allyn and Bacon, London, 1978)
2. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Addison-Wesley Publishing Co., 2003)
3. H. Marshall, *The Theory of Groups*, (Macmillan, 1967)
4. J. A. Gallian, *Contemporary Abstract Algebra*, (Narosa 1998)
5. I.N. Herstein, *Topics in Algebra*, (Xerox Publishing Company Mass, 1972)
6. J. S. Rose, *A Course on Group Theory*, (Dover Publications, 1994)
7. Humphreys, John F. *A Course on Group Theory*, (Oxford University Press, 2004)
8. K. Hoffman, *Linear Algebra*, (Prentice Hall, 1971)
9. I.D. Macdonald, *The Theory of Groups*, (Oxford, Clarendon Press, 1975)

Paper (IV-VI) option (iv): Rings and Modules

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Ring Theory

Construction of new rings
- Direct sums, Polynomial rings
- Matrix rings
- Divisors, units and associates
- Unique factorisation domains

Principal ideal domains and Euclidean domains

Field Extensions

- Algebraic and transcendental elements
- Degree of extension

Algebraic extensions
- Reducible and irreducible
- polynomials Roots of polynomials

Section-II (4/9)

Modules

- Definition and examples
- Submodules
- Homomorphisms
- Quotient modules

Direct sums of modules
- Finitely generated modules
- Torsion modules

Free modules

Basis, Rank and endomorphisms of free modules
- Matrices over rings and their connection with the basis of a free
- module A module as the direct sum of a free and a torsion module

Recommended Books

1. I. N. Herstein, *Topics in Algebra*, (Xerox Publishing Company Mass, 1972)
2. B. Hartley and T. O. Hauvkes, *Rings, Modules and Linear Algebra*, (Chapmann and Hall Ltd., 1970)
3. R. B. Allenly, *Rings, Fields and Groups:An Introduction to Abstract Algebra*, (Edward Arnold, 1985)
4. J. Rose, *A Course on Rings Theory*, (Cambridge University Press, 1978)
5. G. Birkhoff and S. Maclane, *A Survey of Modern Algebra*, (Macmillan, 1964)
6. J. B. Fraleigh, *A First Course in Abstract Algebra*, (Addison-Weseley Publishing Co., 2003)
7. J. A. Gallian, *Contemporary Abstract Algebra*, (Narosa Publisihng House, 1998)
8. K. Hoffman, *Linear Algebra*, (Prentice Hall, 1971)

Paper (IV-VI) option (v): Number Theory

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section- I (5/9)

Congruences

Elementary properties of prime numbers
Residue classes and Euler's function
Linear congruences and congruences of higher
degree Congruences with prime moduli
The theorems of Fermat, Euler and Wilson

Number-Theoretic Functions

Möbius function
The function [x], The symbols O and their basic properties

Primitive roots and indices

Integers belonging to a given exponent (mod p)
Primitive roots and composite moduli
Determination of integers having primitive roots
Indices, Solutions of Higher Congruences by Indices

Diophantine Equations

Equations and Fermat's conjecture for $n = 2$, $n = 4$

Section-II (4/9)

Quadratic Residues

Composite moduli, Legendre symbol
Law of quadratic reciprocity
The Jacobi symbol

Algebraic Number Theory

Polynomials over a field
Divisibility properties of polynomials
Gauss's lemma
The Eisenstein's irreducibility criterion
Symmetric polynomials

Extensions of a field
Algebraic and transcendental numbers
Bases and finite extensions, Properties of finite extensions
Conjugates and discriminants
Algebraic integers in a quadratic field, Integral bases
Units and primes in a quadratic field
Ideals, Arithmetic of ideals in an algebraic number field
The norm of an ideal, Prime ideals

Recommended Books

1. W. J. Leveque, *Topics in Number Theory*, (Vols. I and II, Addison-Wesley Publishing Co., 1961, 1965)
2. Tom M. Apostol, *Introduction to Analytic Number Theory*, (Springer International, 1998)
3. David M. Burton, *Elementary Number Theory*, (McGraw Hill Company, 2007)
4. A. Andrew, *The Theory of Numbers*, (Jones and Barlett Publishers, 1995)
5. Harry Pollard, *The Theory of Algebraic Numbers*, (The Mathematical Association of America, 1975)

Paper (IV-VI) option (vi): Fluid Mechanics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Conservation of Matter

Introduction
Fields and continuum concepts
Lagrangian and Eulerian specifications
Local, Convective and total rates of
change Conservation of mass
Equation of continuity
Boundary conditions

Nature of Forces and Fluid Flow

Surface and body
forces Stress at a point
Viscosity and Newton's viscosity
law Viscous and inviscid flows
Laminar and turbulent flows
Compressible and incompressible flows

Irrotational Fluid Motion

Velocity potential from an irrotational velocity field
Streamlines
Vortex lines and vortex sheets
Kelvin's minimum energy theorem
Conservation of linear momentum

Bernoulli's theorem and its applications
Circulation, Rate of change of circulation (Kelvin's theorem)
Aaxially symmetric motion
Stokes's stream function

Two-dimensional Motion

Stream function
Complex potential and complex velocity, Uniform
flows Sources, Sinks and vortex flows
Flow in a sector
Flow around a sharp edge
Flow due to a doublet

Section-II (4/9)

Two and Three-Dimensional Potential Flows

Circular cylinder without circulation
Circular cylinder with circulation

Blasius theorem
Kutta condition and the flat-plate airfoil
Joukowski airfoil
Vortex motion
Karman's vortex street
Method of images
Velocity potential
Stoke's stream function
Solution of the Potential
equation Uniform flow
Source and sink
Flow due to a doublet

Viscous Flows of Incompressible Fluids

Constitutive equations
Navier-Stokes equations and their exact solutions
Steady unidirectional flow
Poiseuille flow
Couette flow
Flow between rotating cylinders
Stokes' first problem
Stokes' second problem

Approach to Fluid Flow Problems

Similarity from a differential
equation Dimensional analysis
One dimensional, Steady compressible flow

Recommended Book

1. T. Allen and I. L. Ditsworth: *Fluid Mechanics,* (McGraw Hill, 1972)
2. I. G. Currie: *Fundamentals of Mechanics of Fluids,* (CRC, 2002)
3. Chia-Shun Yeh: *Fluid Mechanics: An Introduction to the Theory,* (McGraw Hill, 1974)
4. F. M. White: *Fluid Mechanics,* (McGraw Hill, 2003)
5. R. W. Fox, A. T. McDonald and P. J. Pritchard: *Introduction to Fluid Mechanics,* (John Wiley and Sons Pte. Ltd., 2003)

Paper (IV -VI) optional (vii): Special Relativity and Analytical Dynamics

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

Derivation of Special Relativity

Fundamental concepts
- Einstein's formulation of special
- relativity The Lorentz transformations
- Length contraction, Time dilation and simultaneity
- The velocity addition formulae

Three dimensional Lorentz transformations

The Four-Vector Formulation of Special Relativity

The four-vector formalism
- The Lorentz transformations in 4-vectors
- The Lorentz and Poincare groups
- The null cone structure
- Proper time

Applications of Special Relativity

Relativistic kinematics
- The Doppler shift in relativity
- The Compton effect

Particle scattering

Binding energy, Particle production and particle decay

Electromagnetism in Special Relativity

Review of electromagnetism
- The electric and magnetic field intensities
- The electric current

Maxwell's equations and electromagnetic waves

The four-vector formulation of Maxwell's equations

Section-II (4/9)

Lagrange's Theory of Holonomic and Non-Holonomic Systems

Generalized coordinates
- Holonomic and non-holonomic systems
- D'Alembert's principle, D-delta rule

Lagrange equations

Generalization of Lagrange equations

Quasi-coordinates

Lagrange equations in quasi-coordinates

First integrals of Lagrange equations of motion

Energy integral

Lagrange equations for non-holonomic systems with and without Lagrange multipliers

Hamilton's Principle for non-holonomic systems

Hamilton's Theory

Hamilton's principle

Generalized momenta and phase space

Hamilton's equations

Ignorable coordinates, Routhian function

Derivation of Hamilton's equations from a variational principle

The principle of least action

Canonical Transformations

The equations of canonical transformations

Examples of canonical transformations

The Lagrange and Poisson brackets

Equations of motion, Infinitesimal canonical transformations and conservation theorems in the Poisson bracket formulation

Hamilton-Jacobi Theory

The Hamilton-Jacobi equation for Hamilton's principal function

The harmonic oscillator problem as an example of the Hamilton-Jacobi method The Hamilton-Jacobi equation for Hamilton's characteristic function

Separation of variables in the Hamilton-Jacobi equation

Recommended Books

1. A. Qadir, *An Introduction to Special Theory of Relativity,* (World Scientific, 1989)
2. M. Saleem and M. Rafique, *Special Relativity: Applications to Particle and the Classical Theory of Fields,* (Prentice Hall, 1993)
3. J. Freund, *Special Relativity for Beginners,* (World Scientific, 2008)
4. W. Ringler, *Introduction to Special Relativity,* (Oxford University Press, 1991)
5. H. Goldstein, C.P. Poole and J.L. Safko, *Classical Mechanics,* (Addison-Wesley Publishing Co., 2003)
6. W. Greiner, *Classical Mechanics – Systems of Particles and Hamiltonian Dynamics,* (Springer-Verlag, 2004)
7. E.J. Saletan and J.V. Jose, *Classical Dynamics: A Contemporary Approach*, (Cambridge University Press, 1998)
8. S.T. Thornton and J.B. Marion, *Classical Dynamics of Particles and Systems*, (Brooks Cole, 2003)

Paper (IV-VI) option (viii): Theory of Approximation and Splines

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Euclidean Geometry

Basic concepts of Euclidean geometry
Scalar and vector functions
Barycentric coordinates
Convex hull

Affine maps: Translation, Rotation, Scaling, Reflection and shear

Approximation using Polynomials

Curve Fitting: Least squares line fitting, Least squares power fit, Data linearization method for exponential functions, Nonlinear least-squares method for exponential functions, Transformations for data linearization, Linear least squares, Polynomial fitting

Chebyshev polynomials, Padé approximations

Section-II (5/9)

Parametric Curves (Scalar and Vector Case)

Cubic algebraic form
Cubic Hermite form

Cubic control point form

Bernstein Bezier cubic form
Bernstein Bezier general form
Uniform B-Spline cubic form
Matrix forms of parametric
curves Rational quadratic form

Rational cubic form

Tensor product surface, Bernstein Bezier cubic patch, Quadratic by cubic Bernstein Bezier patch, Bernstein Bezier quartic patch
Properties of Bernstein Bezier form: Convex hull property, Affine invariance property, Variation diminishing property
Algorithms to compute Bernstein Bezier form
Derivation of Uniform B-Spline form

Spline Functions

Introduction to splines
Cubic Hermite splines
End conditions of cubic splines: Clamped conditions, Natural conditions, 2^{nd} Derivative conditions, Periodic conditions, Not a knot conditions

General Splines: Natural splines, Periodic splines

Truncated power function, Representation of spline in terms of truncated power functions, examples

Recommended Books

1. David A. Brannan, *Geometry*, (Cambridge University Press, 1999).
2. Gerald Farin, *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*, (Academic Press. Inc., 2002)
3. John H. Mathews, *Numerical Methods for Mathematics, Science and Engineering*, (Prentice-Hall International Editions, 1992)
4. Steven C. Chapra and Raymond P. Canale, *Numerical Methods for Engineers*, (McGraw Hill International Edition, 1998)
5. Richard H. Bartels, John C. Bealty, and John C. Beatty, *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*, (Morgan Kaufmann Publisher 2006)
6. I. D. Faux, *Computational Geometry for Design and Manufacture*, (Ellis Horwood, 1979)
7. Carl de Boor, *A Practical Guide to Splines*, (Springer Verlag, 2001)
8. Larry L. Schumaker, *Spline Functions: Basic Theory*, (John Wiley and Sons, 1993)

Paper (IV-VI) option (ix): Advanced Functional Analysis

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (4/9)

Compact Normed Spaces

Completion of metric spaces

Completion of normed spaces

Compactification

Nowhere and everywhere dense sets and category

Generated subspaces and closed subspaces

Factor Spaces

Completeness in the factor spaces

Complete Orthonormal set

Complete orthonormal sets

Total orthonormal sets

Parseval's identity

Bessel's inequality

The Specific geometry of Hilbert Spaces

Hilbert spaces

Bases of Hilbert spaces

Cardinality of Hilbert spaces

Linear manifolds and subspaces

Othogonal subspaces of Hilbert spaces

Polynomial bases in L_2 spaces

Section-II (5/9)

Fundamental Theorems

Hahn Banach theorems

Open mapping and closed graph theorems Banach Steinhass theorem

Semi-norms

Semi norms, Locally convex spaces Quasi normed linear spaces

Bounded linear functionals

Hahn Banach theorem

Dual or Conjugate spaces

First and second dual spaces

Second conjugate space of l_p

The Riesz representation theorem for linear functionals on a Hilbert spaces

Conjugate space of $C\ a\ ,b$

A representation theorem for bounded linear functionals on $C\ a,b$

Uniform Boundedness

Weak convergence

The Principle of uniform boundedness

Consequences of the principle of uniform boundedness

Recommended Books

1. G. Bachman and L. Narici, *Functional Analysis*, (Academic Press, New York, 1966)
2. A. E. Taylor, *Functional Analysis*, (John Wiley and Sons, Toppan, 1958)
3. G. Helmberg , *Introduction to Spectral theory in Hilbert spaces*, (N. H. Publishing Company 1969)
4. E. Kreyszig, *Introduction to Functional Analysis with Applications*, (John Wiley and Sons, 2004)
5. F. Riesz and B. Sz. Nagay, *Functional Analysis*, (Dover Publications, New York, Ungar, 1965)

Paper (IV-VI) optional (x): Theory of Optimization

NOTE: Attempt any FIVE questions selecting at least TWO questions from each section.

Section-I (5/9)

The Mathematical Programming Problem

Formal statement of the problem

Types of maxima, the Weierstrass Theorem and the Local-Global theorem Geometry of the problem

Classical Programming

The unconstrained case

The method of Lagrange multipliers

The interpretation of the Lagrange multipliers

Non-linear Programming

The case of no inequality constraints

The Kuhn-Tucker conditions

The Kuhn-Tucker theorem

The interpretation of the Lagrange multipliers

Solution algorithms

Linear Programming

The Dual problems of linear programming

The Lagrangian approach; Existence, Duality and complementary slackness theorems

The interpretation of the dual

The simplex algorithm

Section-II (4/9)

The Control Problem

Formal statement of the problem Some special cases

Types of Control

The Control problem as one of programming in on infinite dimensional space; The generalized Weierstrass theorem

Calculus of Variations

Euler equations

Necessary conditions

Transversality condition

Constraints

Dynamic Programming

The principle of optimality and Bellman's equation

Dynamic programming and the calculus of variations

Dynamic programming solution of multistage optimization problems

Maximum Principle

Co-state variables, The Hamiltonian and the maximum principle The interpretation of the co-state variables

The maximum principle and the calculus of variations

The maximum principle and dynamic programming

Examples

Recommended Books

1. M.D. Intriligator, *A Mathematical Optimization and Economic Theory* (Prentice Hall, 1989)
2. B.S. Gottfried & W. Joel, *Introduction to Optimization Theory*, (Prentice Hall, 1973)
3. R.K. Sudaram, *A First Course in Optimization Theory*, (Cambridge University Press, 1996)
4. S. S. Rao, *Optimization: Theory and Application*, (John Wiley and Sons Ltd, 1978)
5. M. J. Fryer and J. V. Greenman, *Optimization Theory: Applications in Operation Research and Economics*, (Butterworth-Heinemann Ltd, 1987)

6. K. V. Mital and C. Mohan, *Optimization Methods in Operation Research and Systems Analysis*, (New Age Publications, 2005)